Optimal Transformer Tap Selection Using Modified Barrier-Augmented Lagrangian Method

M. M. Adibi, *Life Fellow, IEEE*, R. A. Polyak, I. A. Griva, L. Mili, *Senior Member, IEEE*, and S. Ammari

***Abstract*—The optimal tap selection of transformers directly connected to generators is one of the two significant industry problems. The generator stepup and auxiliary transformers generally are equipped with no-load (fixed) taps that are infrequently changed. Their optimum positions need to be determined for meeting power system discrete states over long periods, covering the annual light-load, peak-load, and emergency conditions. The other and related problem is the use of design reactive capability of generators rather than their actual operating limits. This paper reports on application of a modified barrier-augmented Lagrangian (MBAL)-based nonlinear optimal power-flow method for the optimum selections of the transformer tap positions and the voltage set points of the generators within their over and underexcitation operating limits. The feasibility of the method is demonstrated using a 160-bus test system in operation at a midatlantic utility. It is shown that the method minimizes the deviations of the system bus voltages from the unity while meeting the power system equality and inequality constraints under light- and peak-load conditions.**

***Index Terms*—Generator reactive capability, generator tap selection, optimal power flow.**

I. INTRODUCTION

THE GENERATOR stepup (GSU) and auxiliary (AUX) transformers are generally equipped with no-load (fixed) taps. These taps are infrequently changed and yet they have to meet the continual changing requirements of power system under annual light-load, peak-load, and emergency conditions [1], [2].

The other and related problem is the common use of the generators' design reactive capabilities rather than their actual operating parameters imposed by generator relays and power system constraints. On the other hand, in power system modeling, power-flow programs in use either represent the generator reactive capability by simple high/low limits or by the design ratings, both representations being inadequate, not reflecting the generators' actual operating limits [3], [4].

In this paper, the foregoing issue is formulated as a discrete nonlinear optimization problem, which is solved by means of an optimal power-flow (OPF) method. The latter seeks the optimal GSU and AUX tap positions and the generator voltage set points that minimize the deviations of the system bus voltages from the unity while staying within the generators' under and overexcitation limits and meeting the system light- and peak-load conditions.

Usually, both linear and nonlinear OPFs are solved by means of interior point methods (IPMs) [5]–[11]. These methods resort to the classical barrier function together with the inclusion of slack variables to convert the inequality constraints into equality constraints. For nonlinear OPF, Newton's method is applied [7]–[9]. Despite all of this progress, there is room for improvements in nonlinear OPF methods in general [10] and in IPM methods in particular. While the latter methods produce reasonable results in a number of cases, its associated barrier and penalty methods suffer from serious ill-conditioning problems as reported in [11]. The unbounded increase of the barrier parameter, which is the only tool that allows the control of the computational process in the IPM calculations, leads to the unbounded increase of the condition number of the corresponding Hessian matrix and thereby, results in a rapid shrinkage of the domain of convergence of the Newton's method.

In this paper, the forgoing deficiencies are overcome by means of the modified barrier-augmented Lagrangian (MBAL) method [13], [16]. This method enjoys several interesting features that reveal themselves to be useful in nonlinear OPF. First, thanks to the modified barrier term of the MBAL function, the equality and the inequality constraints are treated in a unified manner without resorting to any slack variables. Second, unlike the IPMs, the Newton-based MBAL algorithm does not require any feasible solution as a starting point. Here, the iteration can be initiated from the flat voltage profile, a solution that violates both the equality and inequality constraints. The discrete aspect of the transformer tap positions is solved by first treating them as continuous variables over their minimum and maximum limits. Then, at the final steps of the algorithm, they are rounded up to their nearest discrete values and the OPF solution is updated. This method gives satisfactory results for transformer taps as indicated in [9].

The MBAL has been successfully applied to the 160-bus test system, which is in operation at a midatlantic utility. This is a closely knit metropolitan type of power system. In contrast to the wide-area type of systems, where concern is with system voltage profile and reactive reserves, in the 160-bus system, concerns were primarily with adequacy of real-power reserve, thermal limits of lines/transformers, and generators' over and underexcitation limits.

The paper is organized as follows: Section II describes the formulation of the optimal transformers tap selection (OTTS) problem as an optimization problem. Section III outlines the

Manuscript received August 13, 2002. This work was supported by the NSF under Grant 0060004.

M. M. Adibi is with IRD Corp., Bethesda, MD 20827 USA (e-mail: madobord@aol.com).

R. A. Polyak and I. A. Griva are with George Mason University, Fairfax, VA 22030 USA (e-mail: Rpolyak@gmu.edu).

L. Mili and S. Ammari are with Virginia Tech, Alexandria, VA 22314 USA (e-mail: lmili@vt.edu).

Digital Object Identifier 10.1109/TPWRS.2002.807093

MBAL algorithm. Section IV describes the test system while Section V provides some of the simulation results.

II. Formulation of the Optimization Problem

Let G, A, and S be sets of all generators, auxiliary buses, and system buses, respectively. Let $B = G \cup A \cup S$ denote the set of all the buses. The objective is to find

— the real-power flows

$$P^p = (P_i^p,\, i \in B), \qquad P^l = (P_i^l,\, i \in B);$$

— the reactive-power flows

$$Q^p = (Q_i^p,\, i \in B), \qquad Q^l = (Q_i^l,\, i \in B);$$

— the bus-voltage magnitudes

$$V^p = (V_i^p,\, i \in B), \qquad V^l = (V_i^l,\, i \in B);$$

— the bus-voltage angles

$$\theta^p = (\theta_i^p,\, i \in B), \qquad \theta^l = (\theta_i^l,\, i \in B);$$

— the transformer tap ratios of the GSU and AUX transformers, which are represented by the superscripts G and A, respectively

$$R_i^G, \qquad i \in G, \qquad R_i^A, \qquad i \in A.$$

All of the variables are in per unit and calculated for both the peak and light load conditions, which are represented by the superscripts p and l, respectively.

The optimal transformer tap selection problem can be formulated as

$$\min \sum_{i \in G} [(V_i^p - 1)^2 + (V_i^l - 1)^2] \tag{1}$$

subject to the following constraints:

— power-flow constraints for the peak and light loads

$$F_p(P^p,\, Q^p,\, V^p,\, \theta^p,\, R^G,\, R^A) = 0;$$
$$F_l(P^l,\, Q^l,\, V^l,\, \theta^l,\, R^G,\, R^A) = 0;$$

— generators capability constraints

$$(P_i^s - P_{i,j}^0)^2 + (Q_i^s - Q_{i,j}^0)^2 \le (d_i^j)^2;$$
$$i \in G;\, s = p,\, l;\, j = 1,\, 2,\, 3;\, Q_i^s \ge a_i P_i^s + b_i,\, i \in G,\, s = p,\, l;$$

— and bounds: $0.95 \le V_i^s \le 1.05$, $i \in G \cup A$, $s = p,\, l$

$$0.90 \le V_i^s \le 1.10, \qquad i \in S,\, s = p,\, l$$
$$P_i^s \ge 0, \qquad i \in G,\, s = p,\, l$$

where $R_i^G \in \mathbf{R}_i^G$, $i \in G$, $R_i^A \in \mathbf{R}_i^A$, $i \in A$, and where the sets $\mathbf{R}_i^G$ and $\mathbf{R}_i^G$ are discrete. Note that at the beginning of the computational process, we relax the discrete constraints by replacing them with the following bounds:

$$R_{i,\min}^G \le R_i^G \le R_{i,\max}^G, \qquad i \in G$$
$$R_{i,\min}^A \le R_i^A \le R_{i,\max}^A, \qquad i \in A.$$

Then, at the final stage, we fix R_i^G and R_i^A at the closest discrete values and refine the solution.

III. MBAL Method for Solving OPF Problems

The idea of combining barrier and penalty function for solving nonlinear optimization problems was suggested more than 30 years ago by A. Fiacco and G. McCormick in [12]. Their sequential unconstrained minimization technique (SUMT) consisted in minimizing the barrier–penalty function followed by the increase of the barrier–penalty parameter. In SUMT, the inequality constraints were treated with log-barrier function and equations with quadratic penalty function. The barrier–penalty parameter was the only tool to control convergence. To guarantee convergence, one has to increase the scaling-penalty parameter to infinity. It leads to the ill-conditioning of the Hessian of the barrier–penalty function and slows the rate of convergence of the algorithm. In particular, it leads to numerical instabilities at the final stage of the computational process and makes it hard to obtain the solution with high accuracy.

Substantial efforts were made during the last 15 years to overcome these difficulties. As a result, the interior points methods (IMPs) were developed. They have become the main direction in modern optimization (see for example [14] and [19]). The main reason is that the IPMs, in general, and primal–dual IPM, in particular, for linear and quadratic programming are very efficient. The situation in nonlinear programming is still not as bright as in linear programming [18]. Therefore, we consider an alternative to the IPM approach, which is based on MBAL theory [13].

Here are a few listed reasons why the MBAL method is suitable for nonlinear programming with both inequality constraints and equations

1) It has a better convergence rate than SUMTs and IPMs under the standard second-order optimality conditions [13].
2) Unlike SUMTs and IPMs, the MBAL method does not require unbounded increase of the barrier–penalty parameter. Therefore, being free from the ill-conditioning effect, the MBAL method keeps stable the area where Newton's method for primal minimization is "well defined." That is, this area does not shrink to a point.
3) The stability of the Newton area makes the MBAL method robust. It allows us to obtain the solution with high accuracy and to observe the "hot start" phenomenon [16]. The latter means that from some point on, only one Newton step is enough to shrink the distance between the current approximation and the solution by any given factor $0 < \gamma < 1$.
4) The MBAL algorithm does not require finding the initial feasible solution. It can be initiated from any starting primal vector and any positive dual vector for the inequality constraints.

Now, let us describe the MBAL function and the correspondent method as applied to the OPF. Let f, c_i and d_j be smooth enough functions, which represent the objective function given by (1) and the inequality and equality constraints, respectively.

Consequently, the OPF formulated in Section II can put in a compact form as

$$x^* \in X^* = \operatorname{Arg\,min}\{f(x) | c_i(x) \geq 0,\ i = 1, \ldots, p; \quad d_j(x) = 0,\ j = 1, \ldots, q\}. \quad (2)$$

Here, x is the control vector containing all of the variables of the OPF problem defined by (1) together with its constraints. By applying the modified barrier function methodology [16] for inequality constraints and by treating the equations with augmented Lagrangian term [17], we obtain the MBAL function, $\mathtt{L}: \Re^n \times \Re^p_+ \times \Re^q \times \Re_+ \to \Re$, which is defined as

$$\mathtt{L}(x, \lambda, v, k) = f(x) - k^{-1} \sum_{i=1}^{p} \lambda_i \ln(k c_i(x) + 1) - \sum_{i=1}^{q} v_i d_i(x) + 0.5k \sum_{i=1}^{q} d_i^2(x). \quad (3)$$

The first two terms in (3) represent the Lagrangian function for the equivalent problems in the absence of the equality constraints since for any fixed $k > 0$, the system $\ln(kc_i(x) + 1) \geq 0, \quad i = 1, \ldots, p$ is equivalent to $c_i(x) \geq 0, i = 1, \ldots, p$. The last two terms represent the augmented Lagrangian for the equality constraints [17]. Along with the classical Lagrangian term $-\sum_{i=1}^{q} v_i d_i(x)$, there is a penalty function $0.5k \sum_{i=1}^{q} d_i^2(x)$, which is designed to penalize the violation of the equality constraints. The modified barrier function given by

$$F(x, \lambda, k) = f(x) - k^{-1} \sum_{i=1}^{p} \lambda_i \ln(kc_i(x) + 1) \quad (4)$$

has all of the characteristics of the interior augmented Lagrangian [16]. Therefore, the MBAL function $\mathtt{L}(x, \lambda, v, k)$ can be viewed as interior–exterior augmented Lagrangian.

Before describing the MBAL-multipliers method, a few important characteristics of the MBAL function at the primal–dual solution should be emphasized. In contrast to the classical barrier function, the modified barrier function exists at the solution together with its derivatives of any order. It is important to emphasize that, unlike SUMTs or IPMs, the convergence of the MBAL is not due to the unbounded increase of the penalty-barrier parameter, but rather due to the Lagrange multiplier update, while the parameter can be fixed.

Let's now describe the MBAL method. For any $x^0 \in \Re^n$, one can find $k > 0$ such that $kc_i(x^0) + 1 > 0$. Therefore, in contrast to IPMs, the numerical realization of the MBAL method does not require finding an initial interior point. The initial dual approximation is not an issue either, so we can take as an initial approximation for the dual vectors $\lambda^0 = e \in \Re^p$ and $v^0 \in \Re^q$, where $e = [1, 1, \ldots 1]^T$. It is assumed that $\ln t = -\infty,\ t \leq 0$ and that (x^s, λ^s, v^s) have been already found. Then, the next approximation of the control vector x is calculated via

$$x^{s+1} = \arg\min\{\mathtt{L}(x, \lambda^s, v^s, k) | x \in \Re^n\} \quad (5)$$

that is

$$x^{s+1}\colon \nabla_x \mathtt{L}(x^{s+1}, \lambda^s, v^s, k) = 0. \quad (6)$$

The new Lagrange multipliers are given by

$$\lambda_i^{s+1} = \lambda_i^s (kc_i(x^{s+1}) + 1)^{-1}, \qquad i = 1, \ldots, p \quad (7)$$

$$v_j^{s+1} = v_j^s - kd_j(x^{s+1}), \qquad j = 1, \ldots, q. \quad (8)$$

Therefore, from (6)–(8), we have

$$\nabla \mathtt{L}(x^{s+1}, \lambda^s, v^s, k) = \nabla f(x^{s+1}) - \sum_{i=1}^{p} \lambda_i^{s+1} \nabla c_i(x^{s+1}) - \sum_{j=1}^{q} \lambda_j^{s+1} \nabla d_j(x^{s+1}) = \nabla_x L(x^{s+1}, \lambda^{s+1}, v^{s+1}) = 0. \quad (9)$$

The convergence of the MBAL method is due to the update of the Lagrange multipliers while $k > 0$ is fixed so that the condition number of the MBAL Hessian remains stable and the convergence domain of the Newton's method for the primal minimization does not shrink to a point. One can expect that the domain where the Newton's method is "well defined" will remain large enough up to the end of the computational process. It makes the computation process robust and, eventually, produces very accurate results. Consequently, the MBAL is an exterior point method for the primal problem (2) because x^s usually does not satisfy neither the primal inequality nor the equations.

It was proven in [13] that both the primal sequence and the dual sequence $\{y^s\} = \{\lambda^s, v^s\}$ converge to the primal–dual solution under the standard second-order optimality conditions. Moreover, the rates of convergence are Q-linear, that is, for primal $\{x^s\}$ and dual $\{y^s\} = \{\lambda^s, v^s\}$ sequences, which were generated by formulas (5)–(8), the following estimation takes place:

$$\|x^{s+1} - x^*\| \leq \frac{c}{k} \|y^s - y^*\|, \|y^{s+1} - y^*\| \leq \frac{c}{k} \|y^s - y^*\| \quad (10)$$

where $c > 0$ is independent of $k \geq k_0 > 0$ and $k_0 > 0$ is large enough (see [13, Theorem 5.1]). The MBAL multipliers method given by (5)–(8) requires solving an unconstrained optimization problem given by (5) at each step.

At this point, we would like to mention that the Newton's method has been used for finding the unconstrained minimizer x^{s+1} starting from x^s. To find the Newton's direction, we solve the system

$$\nabla^2_{xx} \mathtt{L}(x, y^s, k)\xi = -\nabla_x \mathtt{L}(x, y^s, k). \quad (11)$$

The new approximation is given by

$$x := x + t\xi \quad (12)$$

where the steplength $t > 0$ is found using Goldstein–Armijo rule [15]. The process (11) and (12) continues until we reach an approximation $\tilde{x}^{s+1}$ for the primal minimizer x^{s+1} for which the following stopping criteria:

$$\left\|\nabla_x \mathtt{L}\left(\tilde{x}^{s+1}, y^s, k\right)\right\| \leq \frac{\alpha}{k} \left[\left\|\lambda^s \left(kc\left(\tilde{x}^{s+1}\right) + 1\right)^{-1} - \lambda^s\right\| + k\left\|d\left(\tilde{x}^{s+1}\right)\right\|\right] \quad (13)$$

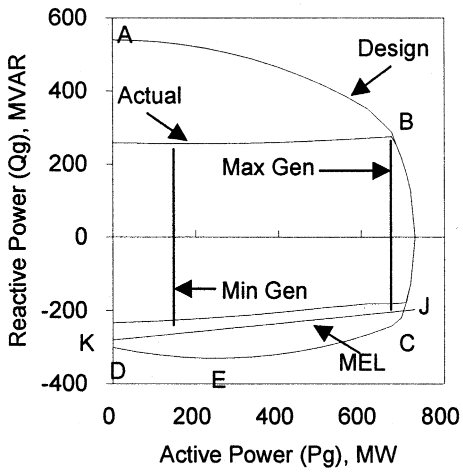

Fig. 1. Generator one-line diagram.

is satisfied while $\alpha > 0$ is given. The corresponding dual variables $\tilde{y}^{s+1} = (\tilde{\lambda}^{s+1}, \tilde{v}^{s+1})$ are found by (7) and (8) when x^{s+1} is replaced by $\tilde{x}^{s+1}$. Then the following bounds hold:

$$\left\|\tilde{x}^{s+1} - x^*\right\| \leq \frac{c(1+\alpha)}{k} \left\|y^s - y^*\right\| \tag{14}$$

$$\left\|\tilde{y}^{s+1} - y^*\right\| \leq \frac{c(1+\alpha)}{k} \left\|y^s - y^*\right\|. \tag{15}$$

Note that the criteria (13) allows us to retain the Q-linear rate of convergence of the MBAL algorithm while, at the same time, it replaces a procedure with infinite steps by a procedure with a finite number of steps.

IV. Test System Description and Objectives

The 160-bus test system consists of 23 generators and 54 loads. The 219 branches include 23 GSU, 23 AUX, and 68 system transformers, all equipped with fixed taps. The 105 overhead and underground lines include 500-, 230-, 138–, 110-, 66-, and 34.5-kV voltages. Of the 23 generators, 15 are steam electric units and eight are combustion turbines. Under peak-load condition, all of the 23 generators are in operation with a total capacity of 5426 MW. Under light-load conditions, the eight combustion turbines are shut down. The minimum generation for the remaining 15 steam electric units is 1129 MW. The peak and light loads are 5101 + j1125 and 1430 + j483 MVA, respectively.

Fig. 1 is a typical one-line diagram for one of the generators. It consists of a 650-MW generator, a system bus at 230 kV, a generator bus at 24 kV, and the auxiliary bus at 4.16 kV. The GSU transformer links the system bus to the generator bus, and the AUX transformer links the generator bus to the AUX bus.

Fig. 2 shows the reactive capability curves for the 650-MW generator. It can be seen that the design reactive capability of the generator is restricted to the area bounded by the A–B–C–E–D. Here, segment AB is limited by the rotor (or field) heating, segment BC is limited by the stator (armature or winding) heating, and segment C–E–D is limited by the armature core-end heating. Line J–K shows limitation imposed by the minimum excitation limit (MEL) relay. The outer curves in Fig. 2 are strictly a function of the generator's design parameters. These curves do not consider the actual operating conditions as the limiting factors. The inner curves show a typical operating limit [2].

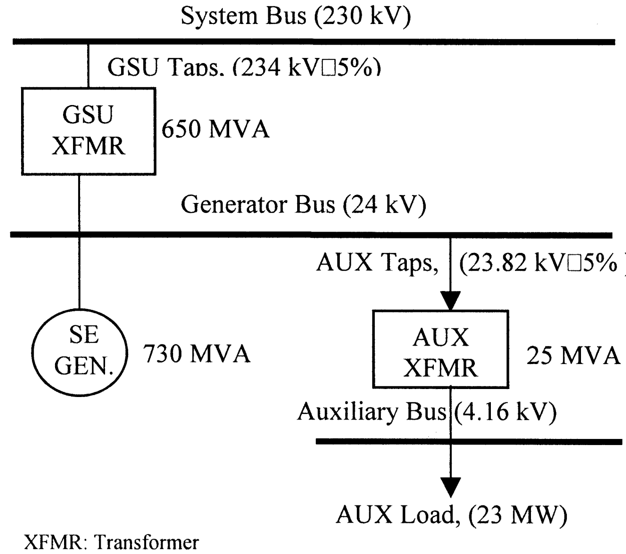

Fig. 2. Generator reactive capability curve.

TABLE I
(a) Generator Reactive Capability Coordinates (b) Gen Reactive Capability Radii and Centers

	SE MW	SE MVAR	CT MW	CT MVAR
A:	0	540	0	174
B:	660	320	192	120
C:	700	230	224	0
E:	250	330	60	118
J:	705	200	212	24
K:	0	280	0	96

Space does not permit the listing of all the 160-bus test system data set and the corresponding MBAL-based OPF results, which are summarized in Section V. Therefore, the input data and the output results are limited to the 650-MW steam electric unit and the 192-MW dual combustion turbine unit.

Table I(a) lists the real- and reactive-power coordinates for the above SE and CT generators. From these coordinates, the radii Ri and centers Opi and Oqi, for arcs A–B, B–C and C–E, are determined as follows:

$$R_1 = N_1/(2\cos\alpha), \qquad N_1 = ((Aq - Bq)^2 + Bp^2)^{1/2},$$

where

$$\alpha = \tan^{-1}(Bp/(Aq{-}Bq)), \qquad O_{P1} = 0.0$$

and

$$O_{q1} = Aq - R_1.$$

$$R_2 = N_2/(2\cos\beta), \quad N_2 = ((Cq - Eq)^2 {+} (Cp{-}Ep)^2)^{1/2}$$

where

$$\beta = \tan^{-1}(Cp - Ep)/(Eq - Cq), \qquad O_{P2} = Ep,$$

and

$$O_{q2} = R_2.$$

$$R_3 = (Bq^2 + Bp^2)^{1/2}, \qquad O_{P3} = 0.0$$

and

$$O_{q3} = 0.0.$$

These Ri, Opi, and Oqi constraints are listed in Table I(b).

TABLE II
GSU AND AUX TRANSFORMER TAP POSITIONS

Steam Electric Unit

Arc	i	Ri	Opi	Oqi
A-B	1	1100	0	-560
B-C	2	1063	250	732
C-E	3	733	0	0

Combustion Turbine Unit

Arc	i	Ri	Opi	Oqi
A-B	1	383.3	0.0	-194.3
B-C	2	1730	60.0	55.0
C-E	3	224	0.0	0.0

TABLE III
PEAK- AND LIGHT-LOAD GENERATION

	SE GSU	SE AUX	CT GSU	CT AUX
Rating,MVA	650	25	250	12
High kV	234	23.82	234	13.8
Low kV	23.4	4.16	13.5	2.3
R %	0.36	0.85	0.04	0.65
X %	11.34	6.57	13.5	6.7
Base kV	230	24	230	13,8
Tap 1	245700	25011	245700	14145
Tap 2	239850	24416	239850	<u>13800</u>
Tap 3	234000	<u>23820</u>	234000	13455
Tap 4	228150	23225	<u>228150</u>	13110
Tap 5	<u>222300</u>	22629	222300	12765
RCF, PU	1.0435	0.9925	1.0400	0.9250

RCF: Ratio Correction Factor

The objective in the application of MBAL-based OPF was to determine a set of discrete tap positions for the 46 GSU and AUX transformers in order to satisfy the following equality and inequality constraints, while minimizing the deviations of system bus voltages from the unity:

- simultaneously meeting both the peak-load (100%) and light-load (28%) conditions;
- staying within the generators' over and underexcitation operating limits during peak- and light-load conditions;
- maintaining all the of the generator bus and auxiliary bus voltages within the $\pm 5\%$ limits;
- maintaining all of the load bus voltages within the $\pm 5\%$ limits;
- keeping all of the system bus voltages within $\pm 10\%$ limits;
- observing all of the other equalities and inequalities considered in a typical power-flow program.

V. TEST SYSTEM RESULTS

Table II lists the ranges of tap positions for the generator stepup and auxiliary transformers for the 650-MW steam electric (SE) unit and the 192-MW combustion turbine (CT) generators. The selected taps positions are underlined. Table III lists the SE and CT generators' real- and reactive–power outputs and operating limits under both peak- and light-load conditions. It can be seen that the generators reactive-power outputs are within the generators operating over and underexcitation limits. Also, observe that the generator bus and auxiliary bus voltages are within the allowed $\pm 5\%$ limits.

Using the Ri, Opi, Oqi,and Jp, q-Kp, q line of Table I(b) for a given power output P, the over and underexcitation limits

TABLE IV
GENERATOR REACTIVE-POWER OUTPUTS AND LIMITS

	MW	MVAR	Voltage
SE Peak Gen.			
Limits:	660.0	389.0	105.0
Generation:	<u>557.6</u>	<u>99.9</u>	<u>105.0</u>
Auxiliary:	23.0	12.1	101.8
SE Light Gen.			
Limits:	165.0	-261.0	95.0
Generation:	<u>165.0</u>	<u>-54.6</u>	<u>97.3</u>
Auxiliary:	13.7	9.1	95.0
CT Peak Gen.*			
Limits	192.0	120.0	105.0
Generation:	<u>192.0</u>	<u>99.6</u>	<u>104.1</u>
Auxiliary:	5.7	3.1	103.9

* Under light-load CTs are shutdown

Q_1, Q_2, Q_3, and Q_4 correspond to the arcs AB, BC, CE, and line JK (MEL), and are expressed as

$$Q_1 = Aq - R_1(1-\cos\delta), \qquad \delta = \sin^{-1}(P/R_1),$$
$$Q_2 = R_2(1-\cos\gamma) - Eq, \qquad \gamma = \sin^{-1}((P-Ep)/R_2)),$$
$$Q_3 = (R_3^2 - P^2)^{1/2},$$

and

$$Q_4 = -Jq - (Kq - Jq)(Jp - P)/Jp.$$

Their values are listed in Table IV. It can be seen that the generator's reactive-power output Q_i is well within the over and underexcitation limits. The other MBAL-based OPF results include the average bus voltage deviations from unity under peak- and light-load conditions, respectively. In particular, we have the following:

- deviations for the 23 generators are 2.8% and 4.6%, well within the 5% limit;
- deviations for the 23 auxiliaries are 2.4% and 3.4%, well within the 5% limit;
- deviations for the 31 loads are 1.3% and 2.2%, also well within the 5% limit.

The performance of the MBAL method for OTTS problem is displayed in Table V. This table gives the objective function f, the norm of the gradient of the Lagrangian $|g|$, the primal–dual gap, infeasibility, and a number of Newton steps per the Lagrange multipliers update. Note that the MBAL algorithm takes 117 steps to converge to the optimal solution, starting from the flat voltage profile. The optimal value of the objective function is 0.2455.

VI. CONCLUSIONS

The MBAL-based OPF has been applied for optimal selection of tap positions for 46 transformers directly connected to the generators. The results have been examined and verified. It has been shown that the MBAL-based OPF method meets the equality and inequality constraints of the 160-bus test system. The method has simultaneously addressed the peak- and light-load conditions while minimizing the deviations of system bus voltages from the unity.

By integrating the generator's reactive constraints with power-flow equations, the MBAL-based OPF has provided an

TABLE V
PERFORMANCE OF THE MBAL METHOD FOR OTTS PROBLEM

	Output	Q1	Q2	Q3	Q4.
SE Peak	100	389	-285	---	-217
SE Light	-55	528	-327	---	-261
CT Peak	99.6	120	-56.8	---	-93.2
CT Light*	---	---	---	---	---

*Out of operation

efficient analytical tool for use in the planning and monitoring of power systems. By being able to address a number of discrete events, such as contingency cases, it has allowed a more secure operation. The MBAL-based OPF can also be used to determine the minimum active and reactive reserve requirement to cope with the loss of the largest generating unit or to prevent voltage collapse within the power system.

In general, the power-flow programs available in the industry can be "cold-started," (i.e., started with bus voltages equal to unity). However, they need to be initialized with generators' transformer tap positions, which are not readily available. By optimal selection of generators' transformer taps, the MBAL-based OPF allows a solution without the need for this initialization. This feature is another significant result that needs to be explored in future research.

The MBAL-based OPF method as shown in the numerical solutions presented before, has produced very encouraging results. In the process, some shortcomings have also been discovered. The main difficulty is related to the first few Lagrange multiplier updates. The necessity to find an approximation for the unconstrained minimizer at each step without well ground stopping criteria makes the initial phase of the computational process time consuming and sometimes difficult. It is important to consider that, generally speaking, the optimal power flows are not convex optimization problems which, in some cases, makes the problem of finding the global unconstrained minimum not an easy task. Therefore, plans are to improve the MBAL code in these two directions.

REFERENCES

[1] M. M. Adibi, D. P. Milaniscz, and T. L. Volkmann, "Remote cranking of steam electric stations," *IEEE Trans. Power Syst.*, vol. 11, pp. 1613–1618, Aug. 1996.

[2] M. M. Adibi, D. P. Milanicz, and T. L. Volkmann, "Simulating transformer taps for remote cranking operations," *IEEE Comput. Applicat. Power*, July 1996.

[3] M. M. Adibi and D. P. Milanicz, "Reactive capability limitation of synchronous machines," *IEEE Trans. Power Syst.*, vol. 9, pp. 29–40, Feb. 1994.

[4] M. M. Adibi, D. P. Milanicz, and T. L. Volkmann, "Optimizing generator reactive power resources," *IEEE Trans. Power Syst.*, vol. 14, pp. 319–324, Feb. 1999.

[5] "Task force on interior-point method applications to power systems," in *IEEE Power Eng. Soc. Mini Lecture, Proc. PICA*, Santa Clara, CA, May 1999.

[6] P. Ristanovic, "Successive linear programming based OPF solution,", IEEE 96 TP111-0.

[7] I. Nejdawi, K. Clements, and P. Davis, "An efficient interior point method for sequential quadratic programming based on optimal power flow," *IEEE Trans. Power Syst.*, vol. 15, pp. 1179–1183, Nov. 2000.

[8] X. Yan and V. H. Quintana, "Improving an interior-point-based OPF by dynamic adjustments of step sizes and tolerances," *IEEE Trans. Power Syst.*, vol. 14, pp. 709–717, May 1999.

[9] W.-H. E. Liu, A. D. Papalexopoulos, and W. F. Tinney, "Discrete shunt controls in a Newton optimal power flow," *IEEE Trans. Power Syst.*, vol. 7, pp. 1509–1518, Nov. 1992.

[10] K. C. Almeida and F. D. Galiana, "Critical cases in optimal power flow," *IEEE Trans. Power Syst.*, vol. 11, pp. 1509–1518, May 1996.

[11] A. Santos, S. Deckman, and S. Soares, "A dual augmented Lagrangian approach for optimal power flow," *IEEE Trans. Power Syst.*, vol. 3, pp. 1020–1025, Aug. 1988.

[12] A. Fiacco and G. McCormick, "Nonlinear programming. Sequential unconstrained minimization techniques," in *SIAM Classic in Applied Mathematics*. Philadelphia, PA: SIAM, 1990.

[13] D. Goldfarb, K. Mints, R. Polyak, and I. Yuzefovich, "Modified barrier-augmented Lagrangian method for constrained minimization," *Comput. Optim. Applicat.*, vol. 14, pp. 55–74, 1999.

[14] Y. E. Nesterov and A. S. Nemirovsky, "Self-concordant functions and polynomial-time methods in convex programming," in *CEMI Academy of Sciences.* Moscow, Russia, 1989.

[15] J. Nocedal and S. Wright, *Numerical Optimization*. New York: Springer-Verlag, 1999.

[16] R. Polyak, "Modified barrier functions," in *Mathematical Programming*. Amsterdam, The Netherlands: North-Holland, 1992, vol. 54, pp. 177–222.

[17] Powell, "A method for nonlinear constraints in minimization problems," in *Optimization*, Fletcher, Ed. London: Academic, 1969, pp. 283–298.

[18] R. Vandebei and D. Shanno, "An interior point algorithm for nonconvex nonlinear programming," *Comput. Optim. Applicat.*, vol. 13, pp. 231–252, 1999.

[19] S. Wright, *Primal–Dual Interior Points Methods*. Philadelphia, PA: SIAM, 1997.

M. M. Adibi (LF'95), received the B.Sc. (honors) degree in electrical engineering from the University of Birmingham, U.K. in 1950, the M.Sc. degree in electrcial engineering from Polytechnic Institute of Brooklyn, New York City, in 1960, and the nuclear engineering degree from the University of Santa Clara, CA, in 1970.

He has spent more than 50 years in the service of electric utilities, assuming various responsibilities, including manufacturing and testing of heavy electrical equipment, operation and maintenance of large thermal power plants, power system planning, and development of computer applications. He has addressed many restoration issues in the 25 IEEE Transaction papers in which he is the principal author.

Mr. Adibi is a Life Fellow of IEEE, a member of CIGRE, and a chartered engineer with the IEE.

R. A. Polyak, received the B.Sc., M.S. (honors), and Ph.D. degrees in mathematics and physics in 1960 from Moscow Central Institute of Mathematics and Economics at the U.S.S.R. Academy of Sciences, Moscow, U.S.S.R.

He was a professor in mathematics with the Continuing Education Graduate School for Engineers and Researchers and a Senior Research Fellow at the Research Institute of Planning at the National Board of Planning in Kiev, U.S.S.R. In 1988, he was a visiting scientist at the Mathematical Sciences Department of IBM Research Center.

Dr. Polyak is a member of the New York Academy of Science. Dr. Polyak joined the faculty of George Mason University in January 1993 as a Research Professor of Operations Research and Engineering. He received Fulbright Scholarship Award in 2001 for his work on the Nonlinear Rescaling Principle in constrained optimization and applications of the NR method to real life problems. He is the author and co-author of six monographs and chapters of books and he has published many papers in refereed professional journals.

I. A. Griva, received the B.Sc. and M.Sc. degrees in applied mathematics from Moscow State University, Moscow, U.S.S.R., in 1993 and 1994, respectively. He received the Ph.D. degree in information technology from George Mason University, Fairfax, VA, in 2002.

Currently, he is a Visiting Assistant Professor at George Mason University. Since 1993, he has been concentrating on theoretical and numerical aspects of nonlinear optimization. The considerable part of his work has been to address practical optimization problems. He has been involved with power-flow optimization since 1998.

Dr. Griva was the recipient of George Mason University Fellowship in 1997-1999 and The Caesarea Edmond Benjamin De Rotschild Foundation award in 2001.

L. Mili (SM'90), received the electrical engineering diploma from the Swiss Federal Institute of Technology, Lausanne, in 1976, and the Ph.D. degree from the University of Liege, Belgium, in 1987.

Currently, he is a Professor of electrical engineering at the Alexandria Research Institite of Virginia Tech. His research interests include optimization methods in power systems, bad data analysis in power system state estimation, power system dynamics and control, vulnerability assessment and mitigation of complex network systems to catastrophic failures, and robust statistics as applied to signal processing.

Dr. Mili is a senior member of the Power Engineering Society of IEEE, the recipient of a 1990 NSF Research Initiation Award, and of the 1992 NSF Young Investigator Award.

S. Ammari, photograph and biography not available at the time of publication.