

An Observability Analysis Method for a Combined Parameter and State Estimation of a Power System

J.B.A London Jr., L. Mili, and N.G. Bretas

Abstract - This paper presents a new observability analysis method of a power system for a combined parameter and state estimation based on the weighted least-squares (WLS) state estimator. In this approach, the state vector is expanded to include both the V- θ state variables and a given set of branch parameters. This augmented state vector is estimated by processing a large collection of measurements recorded over several snapshots of the power system during which the actual system state is quasi static, that is, varies over a small range. This situation typically occurs during the night off-peak periods. The performance of the proposed observability algorithm is evaluated on a small-scale test system.

Keywords – Parameter Estimation, Augmented State Estimation, Observability Analysis, Power Systems.

I. INTRODUCTION

THE state estimator is an essential tool for security monitoring and security analysis of a power system. It aims at estimating in a reliable manner the nodal voltage magnitudes and phase angles of the system by processing a redundant collection of analog measurements. This processing is based on a given mathematical model of the lines and transformers and other transmission devices (i.e. FACTS devices) and on a given topology of the network, which is typically determined from the telemetered signals about the status of switching devices.

Consequently, the state estimator is subject to three types of errors, which are (i) errors on analogical measurements (bad data); (ii) incorrect topology information (topology errors) and (iii) errors on the model of the system equipment (parameter errors). The latter can have adverse impact on the state estimator and are less evident to detect than gross measurements and topology errors.

Despite the importance of the problem, the number of papers devoted to parameter errors is modest as compared to those dealing with gross measurements and topological errors [1]. In the literature, parameter error identification is carried

out either through residual sensitivity analysis [2, 3] or through an augmented state vector. The latter method is of interest to us. Depending on the way that the augmented state is being estimated, we may distinguish two groups of methods: (i) those based on the WLS normal equations [4, 5] and (ii) those that make use of the Kalman filter [6, 7]. The first group of methods suffers from system observability problems since there are rarely enough measurements to enable the estimation of the augmented state vector. To overcome this difficulty, the methods of the second group rely on a transition matrix to predict the state at consecutive scan times through the so-called dynamic equation. However, this transition matrix is usually unknown and hence, has to be identified from real-time measurements, a task rarely considered in the literature. In most of the papers, this matrix is just set equal to the identity matrix, which amounts to assuming that the system state is quasi static.

Taking advantage of the fact that a certain fraction of the measurements remains nearly unchanged during a long period of time, typically during night off-peak hours, we propose to include all these measurements in a single vector and to estimate in a unified manner some branch parameters together with the V- θ state variables. By doing so, the measurement redundancy is significantly increased. Note that during that time period, it is reasonable to assume that the associated state variables are quasi static. In this paper, the observability method developed in [8] is expanded to the augmented state estimation problem.

This paper is organized as follows: Section II revisits the WLS normal equations for the augmented state estimation. Section III outlines the observability method proposed in [8]. Section IV modifies this method to the augmented state while Section V describes some simulation results on a small test system.

II. STATE ESTIMATION BASED ON THE NORMAL EQUATIONS

Power system state estimation is closely related to the statistics regression methods. The non-linear equations relating the (mx1) measurement vector “z” and the (nx1) state vector “x” are:

$$z = h(x) + w \quad (1)$$

where “w” is an (mx1) random noise vector with zero mean jointly Gaussian distribution and $h(\cdot)$ is a vector-valued non-

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linear functions that relates the measurements to the system states. Through the conventional Weighted Least Square (WLS) approach, the state vector 'x' is estimated by recursively forming the Jacobian matrix $H(x) = \partial h(x) / \partial x$, and solving the gain matrix equations:

$$G \Delta x = H^T W (z - h(x)) \quad (2)$$

with $G = H^T W H$, where "W" is the diagonal matrix representing the inverse of the covariance matrix of "w".

III. REVIEW OF THE OBSERVABILITY THEORY BASED ON TRIANGULAR FACTORIZATION AND PATH GRAPH CONCEPTS

Using the linearized state estimator model, and calling G_θ^{DC} the resulting gain matrix, the state estimator will compute the voltage phase angles by solving the equations:

$$G_\theta^{DC} \underline{\theta} = H_{P\theta}^T W_P \underline{Z}_P \quad (3)$$

with $G_\theta^{DC} = H_{P\theta}^T W_P H_{P\theta}$, where: $H_{P\theta}$ is the Jacobian matrix related to active power measurements; Z_P is the measurements vector of active power; W_P the matrix of weighing factors (related to active power measurements);

For observability purposes, what really matters is if the system given by equation (3) does have solution. As the number of measurements is greater than the number of states to be estimated, it is a common practice to attribute zero values for the right side of that equation, that is, one needs to know if the system $G_\theta^{DC} \underline{\theta} = \underline{0}$ does have solution [8-10]. So, observability analysis can be made through the triangular factorization of G_θ^{DC} . In the following, some properties demonstrated in [8-10] will be reviewed.

Property 1: If the system is observable, the factorization of G_θ^{DC} , when a phase angle of the network is not defined as reference, results in only one connected path graph [8] and reduces that matrix to the following form [8-10]:

$$U_\theta^{DC} = \begin{array}{c} 1 \dots \dots \dots nb \\ \begin{array}{|c|} \hline 1 \\ \hline \vdots \\ \hline nb \\ \hline \end{array} \begin{array}{|c|} \hline \text{shaded area} \\ \hline \vdots \\ \hline 0 \\ \hline \end{array} \end{array} \quad (4)$$

Being "nb" the number of buses (the shaded area corresponds to possible non-zero elements).

Property 2: If during the factorization of G_θ^{DC} one Zero Pivot (ZP) appears in the diagonal (i,i), being "i < nb", the

system is not wholly observable and the remaining row and column entries corresponding to that position are zeros [9],[10]. This means the remaining nodes, corresponding to the columns of U_θ^{DC} , from "i+1" to "nb", will be part of other(s) path graph(s) not having path connection(s) with the previous ones [8]. The factorized form of G_θ^{DC} will be [9],[10]:

$$U_\theta^{DC} = \begin{array}{c} \begin{array}{|c|} \hline i \\ \hline \end{array} \begin{array}{|c|} \hline \text{shaded area} \\ \hline \vdots \\ \hline 0 \\ \hline \end{array} \end{array} \quad (5)$$

Remark 1: From the Property 2 one can affirm that the number of zero pivots encountered in the factorization of G_θ^{DC} is equal to the number of path graphs associated with this factorization

Property 3: Observable islands identification: if in the triangular factorization of G_θ^{DC} more than one path graph happens, and:

i) If in the available measurement set there is no injection measurement relating nodes of different path graphs, then the system as a block is unobservable and each subnetwork associated to each isolate path graph constitutes one observable island of the network [8];

ii) If in the available measurement set there are injection measurements relating nodes of different path graphs, then the system as a block is unobservable and it is not possible to assure the subnetworks associate with each isolated path graph constitute observable islands. In order to find observable islands, identify those measurements and discard them (these are irrelevant measurements regarding state estimation to the observable islands). Re-factorize the new matrix.

Repeat the process of discarding and re-factorization up to there is no injection measurement relating nodes of different path graphs. The resulting path graphs will indicate isolate observable islands [8].

IV. DESCRIPTION OF THE PROPOSED APPROACH

The proposed approach comprises three phases:

Phase 1: Determination of the measurement set that will be used to the parameter/state estimation. That is, the measurement set which vary over a small range in a certain time period;

Phase 2: Observability analysis to the proposed state/parameter estimator;

Phase 3: Parameter/state estimation process.

Each one of these phases will be presented in the following.

A. Determination of the measurement set that will be used (Phase 1)

In order to determine the measurement set that will be used to the parameter/state estimation, the first step is to determine the states that vary over a small range in a certain time period. Determining these states, the measurements obtained in all the snapshots, within that time period, that are incident only to these states, will be selected to be used in the parameter/state estimation process.

One state is classified as varying over a small range in a time period if its range in this period was less or equal to 1% (the threshold used to test the convergence of the state estimator).

Remark 2: Before presenting the proposed method to observability analysis, the formulation of the proposed parameter/state estimation will be developed (Phase 3).

B. Proposed parameter/state estimator (Phase 3)

The proposed estimator increases both, the state vector and the measurement vector. The state vector, here called Augmented State Vector (x_{Aug}), is increased with the system parameters to be estimated. Assuming that a subset of the measurements varies over a small range in a certain time period, the measurement vector, here called Augmented Measurement Vector (z_{Aug}), is increased to include the measurements obtained in all snapshots within that time period.

Considering the proposed augmented model, equation (1) becomes:

$$z_{Aug} = h_{Aug}(x_{Aug}) + w_{Aug} \quad (6)$$

The Augmented State Vector " x_{Aug} " is obtained by recursively forming the Augmented Jacobian

Matrix $H_{Aug} = \frac{\partial h_{Aug}}{\partial x_{Aug}}$, and solving the Augmented

Gain Equation:

$$G_{Aug} \Delta x_{Aug} = [H_{Aug}]^t \cdot [W_{Aug}] \cdot [z_{Aug} - h(x_{Aug})] \quad (7)$$

With $G_{Aug} = [H_{Aug}]^t \cdot [W_{Aug}] \cdot [H_{Aug}]$

The parameters to be estimated by the proposed method are the series conductances (G_{km}) and susceptances (B_{km}) of the transmission lines as well as their shunt susceptances (B_{km}^{sh}). Consequently, a system with "L" branches and "nb" buses has $N = 2nb - 1 + 3L$ Augmented States to be estimated (being "nb" voltage magnitude measurements, "nb-1" voltage angles and "3L" parameters).

C. Proposed method to Observability analysis (Phase 2)

As the proposed state/parameter estimator requires the factorization of G_{Aug} , the proposed method to observability analysis is based on the factorization of that matrix and concepts contained in factorization paths. In reality the proposed method is an extension of the method proposed by Bretas [8], that is based on the Gain matrix of the non-augmented model (some of its properties are posed in section III).

Considering the formulation presented in the previous subsection, H_{Aug} has the following structure:

$$H_{Aug} = \begin{bmatrix} H_{P\theta} & H_{Pv} & H_{Pp} \\ H_{Q\theta} & H_{Qv} & H_{Qp} \\ H_{V\theta} & H_{Vv} & H_{Vp} \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial v} & \frac{\partial P}{\partial p} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial v} & \frac{\partial Q}{\partial p} \\ \frac{\partial V}{\partial \theta} & \frac{\partial V}{\partial v} & \frac{\partial V}{\partial p} \end{bmatrix} \quad (8)$$

where: - "P", "Q" and "V" indicate the measurements vector of active and reactive power and voltage measurements respectively; - "θ", "v" and "p" indicates the voltage angle, voltage magnitude and parameter vectors to be estimated respectively. The associated matrix G_{Aug} is:

$$G_{Aug} = [H_{Aug}]^t [W_{Aug}]^{-1} [H_{Aug}] = \begin{bmatrix} G_{\theta} & G_{\theta v} & G_{\theta p} \\ G_{v\theta} & G_v & G_{vp} \\ G_{p\theta} & G_{pv} & G_p \end{bmatrix} \quad (9)$$

Considering this matrix and the theory presented in section III, one can affirm that if the system is observable, considering the proposed augmented model, the triangular factorization of G_{Aug} , when a phase angle of the network as reference is not defined, results in only one ZP and reduces that matrix to the following form:

$$U_{Aug} = \begin{array}{c} \begin{array}{c} \text{nb} \\ \begin{array}{|c|c|} \hline \begin{array}{c} \text{nb} \\ \text{nb} \end{array} & \begin{array}{c} \text{nb} \\ \text{nb} \end{array} \\ \hline \end{array} \\ \end{array} \quad (10)$$

If the system is not wholly observable, the factorization of G_{Aug} results in more than one ZP. In this situation it is necessary to identify the observable islands.

Basing on the theory presented in [8], whose propositions were presented in section III, the identification of the observable islands is made through the following steps:

Step 1: Identify the non-observable parameters. These parameters correspond to the ZPs that appear in the factorization of G_{Aug} from diagonal "2nb" up to "N + 1" (G_{Aug} is a $[(N+1) \times (N+1)]$ matrix);

Step 2: If there is no power measurement relating the non-observable parameters, identify the path graphs associated with the factored submatrix U_0 of U_{Aug} (each isolate path graph constitutes one observable island); stop. Otherwise, go to the next step;

Remark 3: If there is no power measurement relating the non-observable parameters, as these parameters are incident to the branches of the system, one can affirm that there is no power measurement relating nodes of different path graphs. As a consequence, each path graph constitutes, by Proposition 3, one observable island.

Step 3: Remove from the selected measurements those power measurements relating the non-observable parameters (these are discardable measurements regarding parameter/state estimation to the observable islands). Update the factorization of G_{Aug} and return to step 1.

Once determined the observable islands, the parameter/state estimation will be made separately to each one of these islands.

Remark 4: The observability analysis is made through the G_{Aug} matrix obtained in the first iteration.

V. EXAMPLE

Three examples to characterize the proposed approach are presented in the following. In all of them we are considering that the measurements to be used have already been selected.

A. Example 1

The proposed approach is applied to the 3-bus system shown in Fig. 1.

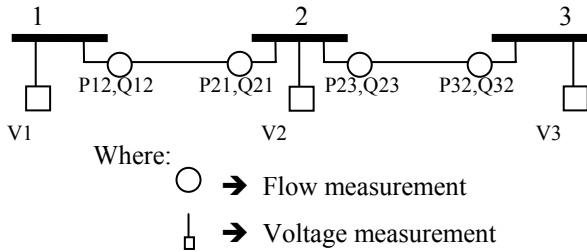


Figure 1. One-line diagram of the test system.

Phase 1: As previously mentioned, let's consider that the measurement set indicated in Fig. 1 were selected in order to be used.

Phase 2: Observability analysis: the corresponding G_{Aug} matrix is obtained and its factorized form is:

$$U_{Aug} = \begin{bmatrix} 01 & 02 & 03 & V1 & V2 & V3 & G12 & G23 & B12 & B23 & B12^{sh} & B23^{sh} \\ 01 & 1 & -1 & 0 & -0.2421 & -0.1989 & 0 & 0.2078 & 0 & 0.3013 & 0 & 0.3013 & 0 \\ 02 & 0 & 1 & -1 & 0 & -0.265 & 0 & 0 & 0.1468 & 0 & 0.1988 & 0 & 0.5265 \\ 03 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ V1 & 0 & 0 & 0 & 1 & -0.4685 & 0 & 0.1384 & 0 & -0.0965 & 0 & -0.2121 & 0 \\ V2 & 0 & 0 & 0 & 0 & 1 & -0.2787 & -0.0554 & 0.0404 & 0.0366 & -0.0331 & 0.1384 & -0.0710 \\ V3 & 0 & 0 & 0 & 0 & 0 & 1 & -0.0234 & -0.0368 & 0.0155 & 0.0299 & 0.0584 & 0.0929 \\ G12 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.1185 & -0.6233 & -0.0973 & -0.8580 & -0.1772 \\ G23 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -0.0252 & -0.7979 & -1.1013 & -1.1696 \\ B12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.0008 & -8.0369 & 0.0113 \\ B23 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.7882 & -20.703 \\ B12^{sh} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.0059 \\ B23^{sh} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

As only one ZP appears, the element $U_{Aug}(03,03)$, the system is observable.

Phase 3: Parameter/state estimation: Considering the measurement set indicated in Fig.1 and using only exact measurements, obtained through a AC power flow, the Augmented State Vector is estimated. The measurement values for that estimation are:

$$Z = \begin{bmatrix} P12 & P21 & P23 & P32 & Q12 & Q21 & Q23 & Q32 & V1 & V2 & V3 \\ 0.3135 & -0.2938 & -0.1055 & 0.1079 & -0.0183 & 0.0771 & -0.0056 & -0.0222 & 1 & 0.9866 & 1 \end{bmatrix}^t$$

Table I shows the results obtained in this phase.

Remark 5: Wrong parameters were used as initial conditions.

Table I – Test 1 results

	Initial conditions	Estimated values	True values
θ1	0	0	0
θ2	0.2	-0.3238	-0.323
θ3	0.3	-0.2049	-0.213
V1	1	1	1
V2	0.8	0.9866	0.9866
V3	0.7	1	1
G12	0.18	0.1918	0.1923
G23	0.17	0.1701	0.1923
B12	-0.8	-0.9591	-0.9615
B23	-0.7	-0.8925	-0.9615
B12 ^{sh}	0.01	0.0201	0.02
B23 ^{sh}	0.015	0.0205	0.02

B. Example 2

The proposed method is again applied to the 3-bus system shown in Fig. 1. But now the selected measurement set selected to be used is: P12, P21, P23, Q12, Q21, Q23, V1, V2 and V3 (without measurements P32 and Q32).

Phase 2: Observability analysis: The associated G_{Aug} matrix is obtained and the factorization results in:

$$U_{Aug} = \begin{bmatrix} 01 & 02 & 03 & V1 & V2 & V3 & G12 & G23 & B12 & B23 & B12^{sh} & B23^{sh} \\ 01 & 1 & -1 & 0 & -0.2421 & -0.1989 & 0 & 0.2078 & 0 & 0.3013 & 0 & 0.3013 & 0 \\ 02 & 0 & 1 & -1 & 0 & -0.265 & 0 & 0 & 0.1468 & 0 & 0.1988 & 0 & 0.5265 \\ 03 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ V1 & 0 & 0 & 0 & 1 & -0.4685 & 0 & 0.1384 & 0 & -0.0965 & 0 & -0.2121 & 0 \\ V2 & 0 & 0 & 0 & 0 & 1 & -0.2000 & -0.0635 & 0.0280 & 0.0420 & -0.0008 & 0.1586 & -0.2105 \\ V3 & 0 & 0 & 0 & 0 & 0 & 1 & -0.0180 & -0.0287 & 0.0119 & 0.0213 & 0.0480 & 0.2164 \\ G12 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.0891 & -0.6225 & -0.0662 & -0.8237 & -0.6708 \\ G23 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -0.0254 & -0.7434 & -1.1102 & -7.5266 \\ B12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -8.0379 & 0 \\ B23 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ B12^{sh} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ B23^{sh} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

As there are three ZPs, elements $U_{Aug}(03,03)$, $U_{Aug}(B23,B23)$ and $U_{Aug}(B23^{sh},B23^{sh})$, the system is not wholly observable. Then it is necessary to identify the observable islands.

Step 1: The non-observable parameters are: B23 and B23^{sh};

Step 2: The power Measurements P23 and Q23 relate the non-observable parameters;

Step 3: P23 and Q23 are removed. The new G_{Aug} is obtained and factored:

$$U_{Aug} = \begin{bmatrix} \theta 1 & \theta 2 & \theta 3 & V1 & V2 & V3 & G12 & G23 & B12 & B23 & B12^{sh} & B23^{sh} \\ \theta 1 & 1 & -1 & 0 & -0.2421 & -0.1929 & 0 & 0.2378 & 0 & 0.3013 & 0 & 0.3013 & 0 \\ \theta 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \theta 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ V1 & 0 & 0 & 0 & 1 & -0.4085 & 0 & 0.1384 & 0 & -0.0965 & 0 & -0.2121 & 0 \\ V2 & 0 & 0 & 0 & 0 & 1 & 0 & -0.0811 & 0 & 0.0536 & 0 & 0.2034 & 0 \\ V3 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ G12 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -0.6202 & 0 & -0.7247 & 0 \\ G23 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ B12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -8.0579 & 0 \\ B23 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ B12^{sh} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ B23^{sh} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

There are five ZPs: $U_{Aug}(\theta 2, \theta 2)$, $U_{Aug}(\theta 3, \theta 3)$, $U_{Aug}(G23, G23)$, $U_{Aug}(B23, B23)$ and $U_{Aug}(B23^{sh}, B23^{sh})$. Return to step 1.

Step 1: The non-observable parameters are: G23, B23 and $B23^{sh}$.

Step 2: As there is no power measurement relating the non-observable parameters, the two path graphs associated with the factorized submatrix U_θ of U_{Aug} constitute two observable islands, being one formed by the buses 1 and 2 and other formed only by the bus 3.

Remark 6: The parameter/state estimation is not processed in case the Observable Island is formed by only one isolate bus.

Phase 3: Parameter/state estimation: Considering the observable island formed by buses 1 and 2, the parameter/state estimation is processed. The measurement values for that estimation are:

$$Z = \begin{bmatrix} P12 & P21 & Q12 & Q21 & V1 & V2 \\ 0.3135 & -0.2938 & -0.0183 & 0.0771 & 0.9866 & 1 \end{bmatrix}^t$$

Table II shows the results obtained in this phase.

Table II – Test 2 results

	Initial conditions	Estimated values	True values
$\theta 1$	0	0	0
$\theta 2$	0.2	-0.3238	-0.332
V1	1	1	1
V2	0.8	0.9866	0.9866
G12	0.18	0.1918	0.1923
B12	-0.8	-0.9591	-0.9615
$B12^{sh}$	0.01	0.0201	0.02

C. Example 3

Let's consider the 3-bus system shown in Fig.1 again. But now the selected measurements are: P12, Q12, P23, Q23, P32, Q32, V1, V2 and V3.

Phase 2: Observability analysis: The factorized form of G_{Aug} matrix is:

$$U_{Aug} = \begin{bmatrix} \theta 1 & \theta 2 & \theta 3 & V1 & V2 & V3 & G12 & G23 & B12 & B23 & B12^{sh} & B23^{sh} \\ \theta 1 & 1 & -1 & 0 & -0.482 & 0 & 0 & 0.2013 & 0 & 0.3557 & 0 & 0.6234 & 0 \\ \theta 2 & 0 & 1 & -1 & 0 & -0.1327 & -0.1284 & 0 & 0.1468 & 0 & 0.1735 & 0 & 0.1735 \\ \theta 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ V1 & 0 & 0 & 0 & 1 & -0.4090 & 0 & 0.1164 & 0 & -0.0659 & 0 & -0.4552 & 0 \\ V2 & 0 & 0 & 0 & 0 & 1 & -0.2833 & -0.0511 & 0.0411 & 0.0289 & -0.0337 & 0.1936 & -0.0721 \\ V3 & 0 & 0 & 0 & 0 & 0 & 1 & -0.0216 & -0.0366 & 0.0122 & 0.0238 & 0.0845 & 0.0925 \\ G12 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.1349 & -0.5659 & -0.1109 & -3.9106 & -0.2088 \\ G23 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -0.7376 & 0 & -1.1662 \\ B12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ B23 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -3.1750 \\ B12^{sh} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ B23^{sh} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

There are two ZPs, elements $U_{Aug}(\theta 3, \theta 3)$ and $U_{Aug}(B12, B12)$. Removing the measurements P12 and Q12, that relating the non-observable parameter B12, the new factorized form of G_{Aug} is:

$$U_{Aug} = \begin{bmatrix} \theta 1 & \theta 2 & \theta 3 & V1 & V2 & V3 & G12 & G23 & B12 & B23 & B12^{sh} & B23^{sh} \\ \theta 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \theta 2 & 0 & 1 & -1 & 0 & -0.1327 & -0.1284 & 0 & 0.1468 & 0 & 0.1735 & 0 & 0.1735 \\ \theta 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ V1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ V2 & 0 & 0 & 0 & 1 & -0.3453 & 0 & 0.0510 & 0 & -0.0410 & 0 & -0.0539 & 0 \\ V3 & 0 & 0 & 0 & 0 & 1 & 0 & -0.0337 & 0 & 0.0274 & 0 & 0.0882 & 0 \\ G12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ G23 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -0.7376 & 0 & -1.1662 \\ B12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ B23 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -3.1750 \\ B12^{sh} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ B23^{sh} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

There are six ZPs, elements $U_{Aug}(\theta 1, \theta 1)$, $U_{Aug}(\theta 3, \theta 3)$, $U_{Aug}(V1, V1)$, $U_{Aug}(G12, G12)$, $U_{Aug}(B12, B12)$ and $U_{Aug}(B12^{sh}, B12^{sh})$. Using path graph concepts one finds two observable islands, one formed by bus 1 and other formed by buses 2 and 3.

Phase 3: Parameter/state estimation: The parameter/state estimator is processed to the island formed by buses 2 and 3. The measurement values for that estimation are:

$$Z = \begin{bmatrix} P23 & P32 & Q23 & Q32 & V2 & V3 \\ -0.1055 & 0.1079 & -0.0056 & -0.0222 & 0.9866 & 1 \end{bmatrix}^t$$

Table III shows the estimation results.

Table III – Test 3 results

	Initial condition	Estimated Values	Real Values
$\theta 2$	0.2	0.2	-0.323
$\theta 3$	0.3	0.3189	-0.213
V2	0.8	0.9866	0.9866
V3	0.7	1	1
G23	0.17	0.1701	0.1923
B23	-0.7	-0.8925	-0.9615
$B23^{sh}$	0.015	0.0205	0.02

Remark 7: Considering the same system, with the wrong parameters, and the same exact measurement sets used in those test, one traditional WLS state estimator was processed. In all the tests, besides to allow the parameter estimation, the voltages and phase angles estimated by the parameter/state

estimator were closer to the true values than those estimated by the traditional WLS state estimator.

VI. CONCLUSIONS

In this paper, an approach to parameter and state estimation based on the WLS normal equations was proposed. This approach augments both the system state vector and the measurement vector. The state vector to be estimated includes some branch parameters together with the bus voltage and phase angles while the measurement vector includes all those measurements that remain nearly unchanged over a period of time. In order to determine the portion of the system whose parameters and states can be estimated through the selected measurements, an observability analysis method was proposed. This method is based on the factorization of G_{Aug} and makes use of the concept of factorization paths. The proposed approach is still in a study phase and this paper presented only the initial results, which are very promising.

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