Iteratively Reweighted Least-Squares State Estimation Through Givens Rotations

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Abstract— This paper presents a robust iteratively reweighted least squares (IRLS) method for power system state estimation (PSSE). An orthogonal implementation through Givens Rotations to solve estimators based on non-quadratic criteria is introduced. The non-quadratic criteria are defined based on the robust statistical theory. The weights associated with the measurements include the effects of measurement variances and the location where each measurement is taken. Bad data identification is a byproduct of the proposed method. After bad data identification, no re-processing is required in order to obtain the correct estimates. The method is validated by using the 14-bus, 30-bus and 118-bus IEEE test systems, in addition to a 340-bus realistic system.

Key-words: Power System State Estimation, Applied Robust Statistics, Robust Iteratively Reweighted Least-Squares, Bad Data Identification.

I. INTRODUCTION

The bad data identification problem in PSSE is usually solved by ranking the normalized residuals, for which the covariance matrix of the residuals is required, [12], [14], [16]. Although the use of the sparse inverse technique developed by Broussole [5] decreases the computational effort, the computing time to perform bad data identification may still be high. Besides, the process of bad data identification requires a previous state estimation, which has to be performed again in order to get correct estimates, even if a single bad measurement is detected and identified.

In addition to the computational aspects raised above, the available methods based on normalized residuals are prone to failure when gross measurements are positional outliers compared to the bulk of measurements. In regression analysis theory, these data are defined as leverage point. This problem was firstly studied by Hampel [9], Huber [11], Rousseeuw *et alii* [19] and later applied in PSSE by Mili *et alii* [15]. In the latter, the authors have concluded that the following situations tend to create leverage points in the estimation process: a) flows and injections associated with relatively short lines; b) flows and injections associated with buses with relatively high number of incident lines. Conventional routines for bad data detection and identification based on normalized residuals, usually fail when gross errors contaminate measurements which are leverage points [15].

This work aims at improving the robustness of the state estimator presented in [15] by including a numerically more stable algorithm to produce correct estimates obtained in the presence of bad measurements. The proposed method combines the numerical robustness of orthogonal methods [21], [22] with the statistical robustness of a particular class of non-quadratic estimators [10] known as *M-estimators* in Statistics [9], [11], [15]. The algorithm derived from those two characteristics is referred to as *Least-Squares Method with Iterative Dynamic Rescaling (LSIDR)*. Re-estimations after bad data identification are not required and the identification of bad measurements (even those classified as bad leverage points) does not demand extra computational effort.

This paper is organized as follows. Section II briefly describes the methodology of improving the robustness of weighted least-squares (WLS) estimator. Section III presents the LSIDR method. Section IV summarizes the implementation of Givens rotation to IRLS method. Section V describes the bad data identification procedure. In section VI, simulations results on the 14-bus, 30-bus and 118-IEEE test systems and a 340-bus realistic system are presented.

II. IMPROVEMENTS TO WEIGHTED LEAST-SQUARES METHOD

Although the WLS is the best estimator in the maximum-likelihood sense when the errors are Gaussian [9], [11], it does not exhibit an inherent capability of filtering bad data. The use of non-quadratic criteria was initially conceived as a means to automatically reject faulty data and still provide good estimates. During the solution steps, measurements which exhibit residuals larger than a pre-

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Table 1: Non-Quadratic Functions

Estimator	*Range*	$\rho(r)$	$\psi(r) = \rho'(r)$	*bep* (β)
WLS	$\Re$	$r^2/2$	r	∞
AQC	$\mid r \mid \leqslant \beta$	$(\beta^2/2)[1-[1-(r/\beta)^2]^3]$	$3r[1-(r/\beta)^2]^2$	4.685
	$\mid r \mid > \beta$	$\beta^2/2$	0	
QC	$\mid r \mid \leqslant \beta$	$r^2/2$	r	2.795
	$\mid r \mid > \beta$	$\beta^2/2$	0	
QSR	$\mid r \mid \leqslant \beta$	$r^2/2$	r	1.264
	$\mid r \mid > \beta$	$2\beta^{3/2}\sqrt{\mid r \mid} - \frac{3}{2}\beta^2$	$\beta^{3/2}\frac{sign(r)}{\sqrt{\mid r \mid}}$	
QT	$\mid r \mid \leqslant \beta$	$r^2/2$	r	1.345
	$\mid r \mid > \beta$	$\beta \mid r \mid -\beta^2/2$	$sign(r).\beta$	

defined break-even point *(bep)* are downweighted; otherwise, they are processed by the quadratic segment of the estimator cost function. In this section, means to control the influence of large residuals are reviewed.

The application of non-quadratic criteria to PSSE was firstly proposed by Merrill and Schweppe [13]. Other contributions can be found in [1], [6], [22], [24]. The estimators investigated in this work are based on Robust Statistics theory and they are described below and summarized in Table 1:

• **Weighted Least-Squares estimator - WLS (Gauss / Legendre, 1795)**, known in regression analysis theory as unbounded estimator.

• **Analytical Quadratic-Constant estimator - AQC (Beaton / Tukey, 1974)**, whose objective function is analytical.

• **Quadratic-Constant estimator - QC (Huber, 1964; Hinich / Talwar, 1975)**, which is similar to the AQC estimator, but with discontinuous $\psi(\cdot)$-function.

• **Quadratic-Squares-Root estimator - QSR (Merrill / Schweppe, 1971)**, which was the first estimator to control the influence of residuals in PSSE.

• **Quadratic-Tangent estimator - QT (Huber, 1964)**, which was the first M-estimator defined on a truly statistical basis. This estimator presents a monotone $\psi(\cdot)$-function.

In Table 1 the variables r and β represent the residual and the break-even point (*bep*) of the non-quadratic function, respectively. This latter was defined assuming an asymptotic efficiency of 95 % [10].

The curves associated with each non-quadratic criterion presented in Table 1 are depicted in Fig. 1. The $\rho(\cdot)$-function is shown in column (a) and its first derivative, $\psi(\cdot)$, in column (b). The curves presented in column (c) are the weighting functions, $q(\cdot)$, used in the robust estimators described in Section III. In the sequel, we restrict attention to the **QT** criterion because, unlike the other non-quadratic criteria, it is defined by a convex $\rho(\cdot)$-function. Hence, the **QT**-objective function has no local minima.

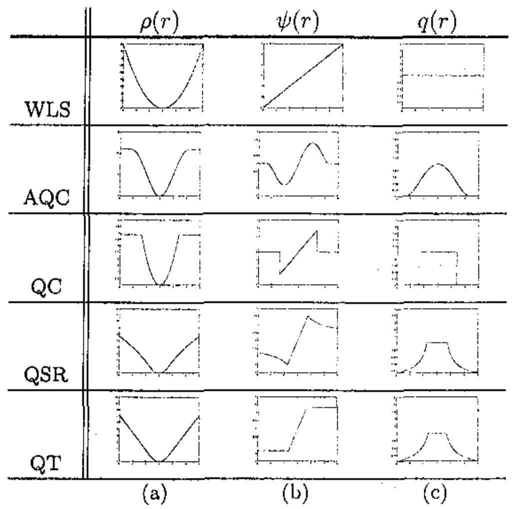

Fig. 1 - Non-quadratic criteria

III. LEAST-SQUARES METHOD WITH ITERATIVE DYNAMIC RESCALING

A. *Residual Weighting Factors*

The *Iteratively Reweighted Least-Squares (IRLS)* method [2] is widely used in linear regression and its convergence properties are well known in Statistics applications [3], [10]. In PSSE, this method was firstly exploited by Mili *et alii* [15]. In this work enhancements to IRLS method is provided through Givens rotations to solve a PSSE problem.

The residuals are incorporated in the estimation model after being standardized as follows:

$$r_{s_i} = \frac{r_i}{\sigma_i . w_i} \tag{1}$$

Here the residual, $r_i = z_i - h_i(x)$, is the difference between the i-th measurement taken in the network (z_i) and the computed values of the corresponding measured quantity ($h_i(x)$). The denominator of equation (1) shows that two distinct weights are used. The first one takes into account the standard deviation (σ_i) of the measurement errors, whereas the second weight (w_i) stems from those measurements classified as leverage points [15].

B. Problem Formulation

Formally, the Schweppe-type state estimation problem considering the leverage points can be written in matrix form as

$$\min J(\hat{\underline{x}}) = \underline{\omega}^T.\rho(\widetilde{\underline{r}}_s).\underline{\omega} \qquad (2)$$

where $\rho(\widetilde{\underline{r}}_s) = diag\{\ \rho(\widetilde{r}_{s_1})\ \ \rho(\widetilde{r}_{s_2})\ \ \cdots\ \ \rho(\widetilde{r}_{s_m})\ \}$ and $\underline{\omega}^T = [\ \omega_1\ \ \omega_2\ \ \cdots\ \ \omega_m\ ]$, represent the non-quadratic function used and the weights related to the leverage points, respectively. The residual vector $\widetilde{\underline{r}}_s$ is defined as:

$$\widetilde{\underline{r}}_s = \xi.\underline{r}_s \qquad (3)$$

where $\xi = (1/Es)$ is a scalar whose purpose is to implement the robust scaling of the standardized residual, and Es is a scale estimator. We propose to use the *median absolute value* to scale the residuals, so that Es is defined as

$$Es = \underset{i}{median}\ \ |\, r_{s_i} |$$

The optimality condition to be fulfilled ($\nabla J(\hat{\underline{x}}) = \underline{0}$) yields

$$\left(\frac{\partial \widetilde{\underline{r}}_s}{\partial \hat{\underline{x}}}\right)^T \cdot \left(\frac{\partial \rho}{\partial \widetilde{\underline{r}}_s}\right).\hat{\omega}^2.e = \underline{0} \qquad (4)$$

where $\hat{\omega} = diag\{\ \omega_1,\ \ \omega_2,\ \ \cdots,\ \ \omega_m\ \}$ and $\hat{\omega}^2.e = [\ \omega_1^2\ \ \omega_2^2\ \ \cdots\ \ \omega_m^2\ ]^T$, and e is a m-dimensional column vector with all entries equal to 1. The residual vector defined in (3) takes the form

$$\widetilde{\underline{r}}_s = \xi.R^{-1/2}.\hat{\omega}^{-1}\underline{r} = \xi.R^{-1/2}.\hat{\omega}^{-1}.(\underline{z} - h(\hat{\underline{x}})) \qquad (5)$$

where $R^{-1/2} = diag\{\ 1/\sigma_1,\ \ 1/\sigma_2,\ \ \cdots,\ \ 1/\sigma_m\ \}$. Using the definitions of $\widetilde{\underline{r}}_s$, $\underline{r}_s$ and e, equation (4) can be re-written as

$$-\xi.H^T(\hat{\underline{x}}).\hat{\omega}^{-1}.R^{-1/2}.\psi(\widetilde{\underline{r}}_s).\hat{\omega}^2.e = \underline{0} \qquad (6)$$

where $\psi(\widetilde{\underline{r}}_s) = (\partial\rho(\widetilde{\underline{r}}_s)/\partial\widetilde{\underline{r}}_s)$. By using the commutative property of diagonal matrices and the assumption that $\xi \neq 0$, the equation above becomes

$$H^T(\hat{\underline{x}}).\hat{\omega}.R^{-1/2}.\psi(\widetilde{\underline{r}}_s).e = \underline{0} \qquad (7)$$

or, defining

$$\widetilde{R_s} = diag\{\ \widetilde{r}_{s_1},\ \ \widetilde{r}_{s_2},\ \ \cdots,\ \ \widetilde{r}_{s_m}\ \} \qquad (8)$$

$$Q(\widetilde{\underline{r}}_s) = \psi(\widetilde{\underline{r}}_s).\widetilde{R_s}^{-1} \qquad (9)$$

and using the fact that $\widetilde{\underline{r}}_s = \widetilde{R_s}.e$ and Eq. (6), yields:

$$H^T(\hat{\underline{x}}).R^{-1}.Q(\widetilde{\underline{r}}_s).(\underline{z} - h(\hat{\underline{x}})) = \underline{0} \qquad (10)$$

The nonlinear vector function $h(\hat{\underline{x}})$ is now approximated by its first order terms of the Taylor expansion about $\hat{\underline{x}}^k$, that is

$$h(\hat{\underline{x}}) \cong h(\hat{\underline{x}}^k) + H(\hat{\underline{x}}^k).\Delta\underline{x} \qquad (11)$$

Using (11), equation (10) becomes

$$H^T(\hat{\underline{x}}).R^{-1}.Q(\widetilde{\underline{r}}_s).\left(\Delta\underline{z} - H(\hat{\underline{x}}^k).\Delta\underline{x}\right) = \underline{0} \qquad (12)$$

or

$$H^T(\hat{\underline{x}}^k).R^{-1}.Q(\widetilde{\underline{r}}_s^k).H(\hat{\underline{x}}^k).\Delta\underline{x} =$$

$$= H^T(\hat{\underline{x}}^k).R^{-1}.Q(\widetilde{\underline{r}}_s^k).\Delta\underline{z} \qquad (13)$$

where $\Delta\underline{z} = \underline{z} - h(\hat{\underline{x}}^k)$.

The solution of the linear system given by equation (13) provides the correction $\Delta\underline{x}$ to the current states $\underline{x}^k$. $\Delta\underline{x}^k$ is obtained through the non-quadratic estimator defined in equation (2) which exhibits the desired characteristics of statistical robustness. The current state is updated as

$$\underline{x}^{k+1} = \underline{x}^k + \Delta\underline{x}^k \qquad (14)$$

It can be easily concluded that Eq. (13) is equivalent to obtaining the solution for the following system of redundant equations:

$$A.\Delta\underline{x} \approx \Delta\underline{y} \qquad (15)$$

where

$$A = R^{-1/2}.Q(\widetilde{\underline{r}}_s^k)^{1/2}.H(\hat{\underline{x}}^k) \qquad (16)$$

$$\Delta\underline{y} = R^{-1/2}.Q(\widetilde{\underline{r}}_s^k)^{1/2}.\Delta\underline{z} \qquad (17)$$

The influence of good leverage points in the estimation process is preserved as follows. Close to the solution, the residuals associated with the good leverage points are within the quadratic segment of the objective function. Therefore, $\psi(\widetilde{\underline{r}}_s) = \widetilde{\underline{r}}_s$, which implies that points with small standardized residuals lead to the cancellation of w_i, so that good leverage points are not downweighted [15]. Thus, accuracy of measurements with Gaussian errors is preserved and the risk of generating critical sets and or critical measurements by inadvertent data elimination is avoided [23].

Large differences among weighting factors applied to the measurements (see Fig. 1.c) may give rise to numerical difficulties in the solution process [4], [8]. One way to alleviate these numerical problems is to solve Eq. (15) through numerically more stable algorithms such as orthogonal methods.

IV. ORTHOGONAL IMPLEMENTATION THROUGH GIVENS ROTATIONS

It is well known that orthogonal methods have been sucessfully applied to solve least-squares problems where the coeficient matrix tends to be numerically ill-conditioned. Givens rotations exhibit the desirable feature of processing the observation matrix by rows, so that it has been the preferred orthogonal method in PSSE applications [17], [20], [21]. Another interesting feature of Givens rotations is the ability of reprocessing information with different weighting factors [7], [18]. This characteristic is advantageously exploited in the orthogonal implementation of the IRLS method as discussed below.

A. Basic Equations

Based on the properties of orthogonal matrices (P is orthogonal *iff* $P^t.P = P.P^t = I$, where I is an identity matrix) and the invariance of Euclidean norm under orthogonal transformations, i.e, $\|P.\underline{y}\| = \|\underline{y}\|$, one can conclude that:

$$\|P.(A.\Delta\underline{x})\| = \|A.\Delta\underline{x}\| \tag{18}$$

$$\|P.\Delta\underline{y})\| = \|\Delta\underline{y}\| \tag{19}$$

If P is defined so that it triangularizes the weighted Jacobian Matrix, that is,

$$P.A = T \tag{20}$$

$$P.\Delta\underline{y} = T_y \tag{21}$$

where

$$T = \left[\frac{T_n}{0}\right] \tag{22}$$

and

$$T_y = \Delta\underline{y} = \left[\frac{\Delta\underline{y}'}{\Delta\underline{y}''}\right] \tag{23}$$

In Eq. (22), T_n is an upper triangular matrix of rank n and 0 is a null matrix of rank $(m - n) \times n$. Similarly, $\Delta\underline{y}'$ and $\Delta\underline{y}''$, Eq. (23), are vectors of dimension n and $(m - n)$, respectively. Finally, the solution of Eq.(15) is achieved through a simple backsubstitution procedure, that is

$$T_n.\Delta\underline{x} = \Delta\underline{y}' \tag{24}$$

B. Givens Rotations Applied to IRLS Method

There are two main advantages in using Givens Rotations in connection with the IRLS method. The first one is numerical: since the IRLS approach is based on the variation of measurement weighting factors through the iterations, there is the possibility that the weights become widely spread, thus increasing the chances of numerical problems in the least-squares solution [4], [8]. It is well documented [7], [17], [20] that, in such situations, orthogonal methods exibit superior numerical performance. The second advantage of employing Givens rotations is the ability of eliminating a measurement in a certain iteration without ruling out the possibility of rescueing it subsequently [18]. In fact, ascribing a zero weight to a measurement in Givens method amounts to discarding its effect on the estimates. However, the same measurement may be re-processed in a subsequent iteration with a nonzero weight. This feature of the orthogonal rotations is particularly advantageous when employed in connection with the so-called "hard redescender" estimators (AQC and QC estimators), since for these estimators measurements whose standardized residuals transiently exceed the break-even point are assigned zero weights. In later iterations, however, it is possible that the residual will lie within the quadratic segment of the cost function, impling that a nonzero weight is to be attributed to the same measurement. This flexibility of switching the measurement weights without requiring refactorizations is due to the reprocessing property of Givens method [7]. The derived algorithm is called *Least Squares method with iterative dynamic rescaling*, or LSIDR for short.

V. BAD DATA IDENTIFICATION

Assuming that the estimated states are acceptable, the following methodology leads to the proper detection and identification of faulty data. After convergence has been achieved the estimation error vector (the difference between the measured and estimated values) should be weighted only by the variance matrix. After that, the residual sum of squares is determined and the $J(\widehat{x})$-test can be performed. If the presence of bad data is detected then identification is provided by weighting the error vector through a robust scale estimator [15]. Finally, the identification procedure can be performed by assuming a suitable value as a threshold.

VI. SIMULATION RESULTS

The results presented below were obtained by keeping the Jacobian matrix constant after two iterations starting at the flat voltage profile. The LSIDR method starts in the second iteration. Except for gross errors, measurements are simulated by adding a random noise, which is within the range of $\pm\ 3\sigma$ to each value obtained from load flow study. The measurement notation is as follows: voltage (V_k); real (P_k) and reactive (Q_k) power injections; real (T_{i-j}) and reactive (U_{i-j}) power flows. The magnitude of the errors is about 20 pu of σ.

The IEEE-14, 30 and 118-bus test systems and a 340-bus realistic system have been used to assess the performance of LSIDR method as applied to PSSE, including the proposed bad data suppression technique. The characteristics of each system and the metering configuration are summarized in Table 2.

The weights corresponding to leverage points are computed in the first iteration. For the 340-bus system, this routine takes about 13 seconds to calculate the projection statistics when it is carried out using a Pentium 133 MHz CPU. Although this computing time is not too high and indicates the possibility of real-time computation, the projection statistics determination can be performed *off-line* [15]. All metering configurations presented above are designed to avoid

Table 2: Test Systems

# of Buses	14	30	118	340
# of lines	20	41	179	494
# of flow meas. (*)	23	45	221	580
# of injection meas. (*)	11	30	65	233
# of voltage meas.	5	30	65	159
total # of meas.	73	180	637	1785
# of leverage points	8	26	20	172

Flow/Inj. meas.(*) are taken in active/reactive pairs.

the formation of critical measurements and sets. The average global redundancy is about 3.1.

To illustrate the performance of an M-estimator implemented through Givens rotations, results obtained through the QT estimator described in Section II are compared with similar results provided by the WLS estimator. Table 3 presents eleven cases carried out on the test systems.

Cases 1, 2 and 3 were simulated on the IEEE-14 bus system. In case 1 the BDI routine of WLS estimator points out only the flow measurement T_{1-2} as bad data, whereas the QT estimator performs the correct identification. In case 2 the WLS fails again and misidentifies the injection P_1. This problem is due to the leverage effect on the branch (made ten times shorter) which connects the buses 1 to 2. On the other hand, the QT estimator presents better estimates and identifies the gross errors correctly. In case 3, five bad data are simulated, among which measurements T_{5-2} and U_{5-4} are leverage points. Voltage measurement (V_8) is not identified by the BDI routine in WLS estimator, whereas the QT estimator again performs better than WLS.

Cases 4 and 5 are carried out on the IEEE-30 bus system. The fourth case is based on perfect measurements, whereas in the fifth case random noise is added to all measurements. In both cases, the WLS estimator fails because of the leverage effect in branch 4-6. Although P_4 is correctly identified, the U_{6-4} and Q_6 are identified instead of U_{4-6} and Q_4, respectively. The QT estimator provides correct estimates and identification regardless of the presence of noise in the measurements.

Cases 6 and 7 were simulated on the IEEE-118 bus system. Local redundancy in case 6 is lower than in case 7. It should be noted that WLS performs correctly in the latter case, but no bad data are identified in case 6. The QT estimator works better no matter the local level of redundancy. Notice that the bad data are not leverage points.

Cases 8 to 11 were simulated on the 340-bus realistic system. Again, the QT estimator performs better than WLS estimator in all cases. The same kind of misidentification already observed is provided by the WLS estimator. For instance, in case 8 the injection P_{72} is identified instead of P_{118}, whereas in case 9

Table 3: Test Cases

#	Bad Data	True	Meas.	WLS	QT
1	T_{1-2}	1.145	1.460	1.135	1.152
	U_{1-2}	4.939	5.949	5.198	4.943
2	T_{2-1}	-1.123	-.8224	-.9240	-1.161
	U_{5-4}	.1344	.3362	.1369	.1392
	P_2	.1830	.3863	.3862	.1831
3	V_8	1.090	1.120	1.119	1.091
	T_{5-2}	-.3641	-.1513	-.3635	-.3622
	T_{10-11}	-.1271	.0745	-.1257	-.1261
	U_{5-4}	.1344	.3362	.0431	.1339
	P_{13}	-.1350	.0668	-.1356	-.1314
4	U_{4-6}	-.0114	.1887	.1882	-.0113
	P_4	-.0760	.1246	-.0758	-.0756
	Q_4	-.0160	.1840	.1845	-.0156
5	U_{4-6}	-.0114	.1887	.1909	-.0114
	P_4	-.0760	.1246	-.0644	-.0771
	Q_4	-.0160	.1840	.1818	-.0377
6	$T_{115-114}$	-.1093	.0919	.0904	-.1089
	$U_{115-114}$	.0333	.2334	.2317	.0294
7	$T_{115-114}$	-.1093	.0919	-.1097	-.1115
	$U_{115-114}$	.0333	.2334	.0350	.0325
8	T_{118-72}	3.085	3.734	3.262	3.087
	P_{118}	.0000	.2000	.2031	.0019
9	P_{72}	-.4470	-.2279	-.2669	-.4495
	Q_{72}	-.2920	-.0836	-.1598	-.2919
10	P_{118}	.0000	.2000	-.0092	.0314
	Q_{118}	.0000	.2000	.1817	.00004
	U_{118-72}	-.0050	.1950	.1445	-.0051
11	$T_{282-250}$	.4772	.6988	.6237	.4996
	$T_{282-251}$	.4772	.6988	.6237	.5006
	$U_{282-250}$	-.1861	.0173	-.1022	-.1861
	P_{282}	.0000	.2000	.2447	.0046

P_{118} is misidentified instead of P_{72}. In case 10, only the injection P_{118} is identified by the BDI routine of the WLS estimator. In the last case, the QT estimator performs better than WLS, in spite of the strong correlated bad data.

Table 4 presents some performance indices such as the residual sum of squares (RSS), number of iterations (ITER), average absolute error (AAE) of the estimates and the corresponding standard deviations (SDE) for all cases presented in Table 3. It should be emphasized that the RSS provided by QT estimator takes into account the weighting factor $\psi(r_{s_i})/r_{s_i}$ in addition to the weight based on measurements variances. This explains the low values for RSS, since measurements with large residuals are heavily downweighted.

VII. CONCLUSIONS

A statistically and numerically robust state estimator is introduced in this paper. The proposed estimator is refered to as *LSIDR* and is capable of providing valid estimates with no need of measurement re-processing when there are gross errors in the set of data. The advantage of the LSIDR algorithm as compared to others is that, upon convergence, the suspect measurements may be deleted and one iteration carried out to decrease the bias in the estimates. In order not to penalize good leverage points, the existence of a quadratic segment in the objective function should be ensured. In addition to being statistically robust, the LSIDR estimator is also a numerically more stable algorithm because it is based on orthogonal transformations.

Table 4: Performance Indices

a)	WLS			
Case	RSS	ITER	AAE	SDE
1	307.3	3	.03807	.09428
2	4.922	3	.01239	.04564
3	12.01	3	.01538	.06191
4	.0205	4	.00449	.02954
5	25.39	4	.00627	.02975
6	42.74	4	.00345	.01490
7	30.67	4	.00250	.00407
8	11205.	6	.00278	.03027
9	11030.	6	.00329	.03023
10	10984.	6	.00347	.03066
11	11327.	6	.00318	.03110

b)	QT			
Case	RSS	ITER	AAE	SDE
1	1.093	3	.03004	.08116
2	2.048	3	.00376	.00903
3	10.52	8	.00251	.00588
4	.0033	4	.00005	.00012
5	7.901	4	.00257	.00709
6	6.227	5	.00478	.01199
7	7.884	5	.00411	.00856
8	150.33	5	.00235	.03843
9	149.86	5	.00235	.03840
10	43.731	6	.00265	.04555
11	45.337	6	.00269	.04574

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BIOGRAPHIES

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Discussion

Ibrahim O. Habiballah (King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia): I wish to commend the authors for their detailed description of the new iteratively reweighted least squares approach to improve the robustness of the power system state estimator (PSSE) by including a numerically more stable algorithm "givens rotation" in order to produce correct estimates.

I would appreciate the authors' comment on the following points:

1) How robust is the proposed method compared to another similar iteratively reweighted method that has been presented in [1] which also claims that no re-processing is required in order to obtain the correct estimates ?
2) The RSS index in Table 4 provides no valuable information for the comparison purpose.
3) Does the AAE index of Table 4 compare the average absolute error between the estimated and measured values or the average absolute error between the estimated and true values ? I have the feeling that the large values of cases 6 & 7 in Table 4 using this index with the QT method when compared with the WLS method is due to the use of the former way of defining the AAE index.
4) Is the number of iterations of the WLS method in Table 4 accounted for the re-processing required in order to obtain the correct estimates ?
5) It is believed that more problems can be anticipated on systems with lower redundancy (say, less that 2.0). Did the authors try such cases ?

[1] M.M. Smith, R.S. Powell, M.R. Irving, and M.J.H. Sterling, *"Robust Algorithm for State Estimation in Electrical Networks"*, **IEE Proceedings-C**, Vol. 138, No. 4, pp. 283-288, July 1991.

Robson C. Pires, Antonio Simões Costa, and Lamine Mili: We would like to thank the discusser for his interest in our paper and for his pertinent questions. We will answer the questions in the same order as they have been raised:

The estimator proposed in [A] is also based on a reweighting scheme capable of emulating a least absolute value (LAV) estimator from a least-squares framework. As in our work, that property is exploited in order to devise a robust estimator. However, there are major differences between the two approaches, namely:

a) Our method stresses the treatment of leverage points, so that bad leverage points are rejected while the beneficial effects of good leverage points on the estimates are preserved. The mechanism through which this is accomplished is described in the text following Eq. (17);
b) Our use of Givens rotations, which is motivated by their numerical robustness. Good numerical behaviour is relevant when reweighting is performed, since it is well known that weights too spread apart may be the source of convergence difficulties in least-squares problems. In [A], the authors cope with that problem by using heuristic procedures to control the magnitude of the weights.

The discusser's second question gives us the opportunity to clarify that the WLS estimator employed in our simulations is also based on Givens rotations. This WLS orthogonal estimator allows that bad data detection, identification and rejection be performed within the same iterative process that computes the state estimates [18]. As a consequence, the RSS values presented in Table 4, part (a), do not include the effects of the gross measurements that the orthogonal WLS implementation was able to succesfully identify and reject in the course of the iterations. This explains some RSS values for the WLS estimator that may seem low, if one expected that no bad data processing had taken place at all.

Nevertheless, the RSS values do look meaningful for comparison purposes for most cases in Table 4. In Case 1, for instance, for which the chi-square threshold for RSS is 61.8, the value shown in part (a) of Table 4 clearly reveals that the WLS estimator was not able to reject all simulated bad data. On the other hand, the corresponding result in part (b) of the table shows that the QT estimator provides bad data-free estimates. The same also applies to cases 8 through 11. This pattern does not repeat itself for all cases, however, due to the fact that the rejection of misidentified data connected with the WLS estimator sometimes produces very low local redundancy levels. In some cases, this leads to situations where non-identified bad measurements become critical (see Case 2), and consequently the values of the RSS index does not reveal their presence.

Concerning the AAE index presented in Table 4, it compares the average absolute error between the estimated and true values.

Since, as explained above, bad data processing for the WLS estimator is a built-in procedure performed within the iterative process, the number of iterations reported in the paper accounts for all required measurement re-processing.

The influence of low local redundancy on the performance of both WLS and QT approaches are illustrated in Cases 6 and 7 of the paper. In Case 7, the level of local redundancy is significantly higher than in Case 6, so that both methods perform well. For the low local redundancy situation of Case 6, however, the WLS estimator fails, as shown in Table 3, while the QT estimator is still able to properly identify the simulated bad measurements and produce good estimates for the measured quantities.